

MATH: NUMBER RECOGNITION AND SUBITIZING PRESENTER NOTES

This guide walks you through presenting the Math: Number Recognition and Subitizing in-service suite for American Indian and Alaska Native (AIAN) programs. This in-service suite includes PowerPoint slides and supporting materials. The main PowerPoint presentation and optional slides were developed for all professional audiences (teachers, home visitors, family child care providers) working in AIAN programs or supporting children AIAN children and families. Additionally, separate PowerPoint slides were developed for use with home visitors. Learning activities, tip sheets, and activity sheets are labeled for their intended audiences.

MATERIALS NEEDED:

- Presenter PowerPoint slides
- Introductory video (found on the ECLKC website)—play at the start of your training session, if internet connection is available
- Projector and audio equipment
- Internet connection if you plan to show optional video examples

BEFORE YOU BEGIN:

- This presentation provides participants with information to support their understanding and teaching of math concepts, particularly number recognition and subitizing.
- Encourage participants to view this presentation and think about how it pertains to AIAN children in their centers or home-based settings.
- Throughout this presentation, examples describe how children in AIAN programs naturally engage in math activities every day while they participate in their cultural and language activities.
- The presentation includes video clips of teachers fostering number recognition and subitizing skills and children engaged in number thinking as they play. Although most of the videos are teacher examples, home visitors can watch the strategies and use them for their own professional development and to encourage families to replicate.
- Optional learning activities offer participants opportunities to understand how number recognition and subitizing develops and think about strategies that support children's development in this area whether in the classroom, home, or community.
- Adaptation boxes individualize this training for your specific training group.

LIST OF AVAILABLE CONTENT

HANDOUTS

- What is Culture?
- Multicultural Principles for Early Childhood Leaders
- Number Recognition and Subitizing Stretches Across ELOF Domains
- Making It Work Cycle
- Tips for Families: Number Recognition and Subitizing
- Tips for Education Staff: Learning Trajectories of Number Recognition and Subitizing
- Tips for Home Visitors: Learning Trajectories of Number Recognition and Subitizing
- Activities for Education Staff: Fantastic Five
- Tips for Education Staff: Supporting Families in Math Learning
- Activities for Families: Help Your Child Learn



- Tips for Home Visitors: Supporting Families in Math Learning
- Learning Activity for Education Staff: Subitizing Throughout the Day
- Activities for Families: Brainstorming
- Learning Activity for Home Visitors: Brainstorming
- Helpful Resources
- Tools for Supervisors: Number Recognition and Subitizing

OPTIONAL HANDOUTS

Head Start Performance Standards Supports Writing, Culture, and Language

OPTIONAL SLIDES

- Optional Slide 1: HSPPS Support Math
- Optional Slide 2: HSPPS Supports Language and Culture
- Optional Slide 3: HSPPS Supports Language and Culture
- Optional Slide 4: HSPPS Supports Math
- Optional Slide 5: Sub-Domain-Emergent Mathematical Thinking: Infant Toddler
- Optional Slide 6: Sub-Domain-Emergent Mathematical Thinking: Preschool
- Optional Slide 7: LT2 Level: Foundation
- Optional Slide 8: LT2 Level: Small Collection Namer
- Optional Slide 9: LT2 Level: Maker of Small Collections
- Optional Slide 10: LT2 Level: Perceptual Subitizer to 4
- Optional Slide 11: LT2 Level: Perceptual Subitizer to 5
- Optional Slide 12: LT2 Level: Conceptual Subitizer to 5
- Optional Slide 13: LT2 Level: Conceptual Subitizer to 10
- Optional Slide 14: Creating a Learning Environment
- Optional Slide 15: Learning and Teaching with Learning Trajectories (LT2)

OPTIONAL SLIDES FOR HOME-BASED CARE (HOME VISITORS) ONLY

- HV Optional Slide 1: Introduction
- HV Optional Slide 2: HSPPS Support Math
- HV Optional Slide 3: HSPPS Support Language and Culture
- HV Optional Slide 4: HSPPS Support Language and Culture
- HV Optional Slide 5: HSPPS Support Math—Home-Based Programs
- HV Optional Slide 6: Center-Based and Family Child Care Practices
- HV Optional Slide 7: Home Visiting Practices
- HV Optional Slide 8: What's Different for a Home Visitor?
- HV Optional Slide 9: Theory of Change for Home-Based
- HV Optional Slide 10: Support Parents in Speaking Math
- HV Optional Slide 11: Home Visitors Can Support Children's Subitizing



PRESENTATION OUTLINE

SLI	DE	HANDOUTS NEEDED	HOME VISITOR POWERPOINT ADAPTATION	HOME VISITOR HANDOUTS	OPTIONAL SLIDES	OPTIONAL HANDOUTS
1.	Math Number Recognition and Subitizing Introduction		Replace with HV Optional Slide 1			
2.	Every Individual is Rooted in Culture-	 What is Culture? Multicultural Principles for Early Childhood Leaders 				
3.	Two-Eyed Seeing-					
4.	Math Stretches Across ELOF Domains					
5.	Math Stretches Across	 HSPPS Support Math, Culture, and Language Number Recognition and Subitizing Stretches Across ELOF Domains 	 HV Optional Slide 2 HV Optional Slide 3 HV Optional Slide 4 HV Optional Slide 5 		 Optional Slide 1 Optional Slide 2 Optional Slide 3 Optional Slide 4 Optional Slide 5 Optional Slide 6 	
6.	Making It Work	 Making It Work Cycle 				
7.	Language and Culture Matter					
8.	House Framework for Effective Teaching Practices					
9.	Parent, Family, and Community Engagement					
10.	House Framework for Effective Teaching Practices					
11.	Culturally and Linguistically Responsive Practices					



SLIDE	HANDOUTS NEEDED	HOME VISITOR POWERPOINT ADAPTATION	HOME VISITOR HANDOUTS	OPTIONAL SLIDES	OPTIONAL HANDOUTS
12. Session Objectives		 Insert HV Optional Slides 6-9 after Objectives 			
13. Subitizing					
14. Some Examples of Subitizing in the Classroom					
15. Learning Trajectory					
16. Part 1 of the Learning Trajectory					
17. Learning Trajectory Goal for Number Rec/ Subitizing	 Tips for Families: Number Recognition and Subitizing 				
18. Not This					
19. Number Recognition					
20. Why Is This Goal Important					
21. Part 2 of the Learning Trajectory					
22. Young Children and Number					
23. Habituation Simulation					
24. 2 Yellow Triangles					
25. Perceptual Subitizing					
26. What Did you See? 4 Yellow Cubes					
27. Conceptual Subitizing					
28. What Do You See (8 Cubes)?					

SLIDE	HANDOUTS NEEDED	HOME VISITOR POWERPOINT ADAPTATION	HOME VISITOR HANDOUTS	OPTIONAL SLIDES	OPTIONAL HANDOUTS
29. What Did You See?					
30. LT Level: Foundations	Tips for ES: Learning Trajectories		Tips for HV: Learning Trajectories	Insert Optional Slide 7	
31. LT Level: Small Collection Namer				• Insert Optional Slide 8	
32. LT Level: Maker of Small Collections				Insert Optional Slide 9	
33. LT Level: Perceptual Subitizer to 4				 Insert Optional Slide 10 	
34. LT Level: Perceptual Subitizer to 5				 Insert Optional Slide 11 	
35. LT Level: Conceptual Subitizer to 5				• Insert Optional Slide 12	
36. LT Level: Conceptual Subitizer to 10				 Insert Optional Slide 13 	
37. Part 3 of the Learning Trajectory					
38. Math Language		Insert HV Optional Slide 10		• Insert Optional Slide 14	
39. Support Subitizing for AI/AN Children	 Activities for ES: Fantastic Five Tips for ES: Supporting Families with Math Learning Activities for Families: Help Your Child Learn 	Insert HV Optional 11	 Tips for HV: Supporting Families with Math Learning Activities for Families: Help Your Child Learn 		

SLIDE	HANDOUTS NEEDED	HOME VISITOR POWERPOINT ADAPTATION	HOME VISITOR HANDOUTS	OPTIONAL SLIDES	OPTIONAL HANDOUTS
40. What to Do?	 Learning Activity for ES: Subitizing Throughout the Day Activities for Families: Brainstorming 		 Learning Activity for HV: Subitizing Throughout the Day Activities for Families: Brainstorming 		
41. Rhythmic Subitizing					
42. Supporting Children with Suspected Delays or Identified Disabilities					
43. Children with Disabilities					
44. Review	Helpful ResourcesTools for Supervisors			• Insert Optional Slide 15	

ES: Indicates Tips/Activity Sheet for Education Staff

HV: Indicates Tips/Activity Sheet for Home Visitors

F: Indicates Tips/Activity Sheet for Families



SLIDE 1:

WELCOME AND INTRODUCTIONS:

- Begin the training by giving participants background information about yourself.
- Provide an opportunity for participants to introduce themselves.
- Encourage them to use their tribal language to introduce themselves.

INTRODUCE THE TOPIC:

Begin with an activity to get participants thinking about how their culture naturally uses math (i.e., counting beads, making baskets, planting, preparing for winter, etc.)

DISCUSSION: HAVE PARTICIPANTS SHARE WHAT MATH SKILLS THEY USED THAT MORNING.

Offer examples and try to use examples that include local culture and language. Emphasize—you use math every day! The Native way of life is rich with math.

REFERENCES

- Clements, D. H. 1999. "Subitizing: What is it? Why Teach It?" Teaching Children Mathematics, 5(7), 400–405.
- Clements, D. H., J. Sarama, & B. L. MacDonald. 2017. "Subitizing: The Neglected Qualifier." In N. Anderson & M. W. Alibali (Eds.), Constructing Number: Merging Perspectives from Psychology and Mathematics Education: Springer.

ADAPTATION FOR HOME VISITORS:

Replace with HV Optional Slide 1



SLIDE 2:

As we explore number recognition and subitizing, it is important to recognize that culture influences beliefs and behaviors. As cited in Multicultural Principles for Early Childhood Leaders, Principle #1, "culture influences every aspect of human development and is reflected in childrearing beliefs and practices." Let's define culture and discuss how it influences the way we present and learn these concepts in this in-service suite.

Culture is:

- A set of values, knowledge, and traditions.
- Always changing and evolving.
- Language, art, morals, and customs.
- Child rearing and educational practices.

Culture is important because:

- It links the home and classroom environments.
- It enhances children's learning experiences.
- It affects the way children learn.
- It influences what families and communities think is important.

Provide time to discuss culture and their definitions.

RESOURCE

Multicultural Principles for Early Childhood Leaders, Learning Extension Principle 1: Every Individual is Rooted in Culture:

https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/multicultural-principles-learning-extensions-principle-01.pdf

MATERIALS NEEDED:

- Handout: What is Culture?
- Handout: Multicultural Principles for Early Childhood Leaders





Two-Eyed Seeing



"Two-Eyed Seeing asks us to see our strengths, the best in our ways of knowing, while also asking us to respect and celebrate our differences. Two-Eyed Seeing acknowledges the necessity of formal structure yet that it must be preamble to and receptive of new understandings and opportunities, i.e., understandings associated with 'Spirit of the East' which brings the 'gift of newness, of transformation.'"



SLIDE 3:

Throughout this suite we are learning how to use the ELOF and developmentally appropriate practices to teach math to young children. Many programs use the ELOF and/or their state's early learning guidelines. AIAN culture is rich with opportunities to learn the math skills presented in this suite.

The descriptive label Two-Eyed Seeing seeks to avoid portraying the situation as a "clash of knowledges" or as contributing to "knowledge domination or assimilation." We recognize that in one set of circumstances we may choose to call upon the strengths within Indigenous science, and Western Science in another set of circumstances. Thus, Two-Eyed Seeing will often require a "weaving back and forth" between these two perspectives.

It intentionally and respectfully brings together our different knowledges to motivate people, Aboriginal*, or non-Aboriginal alike to leave the world in a better place. This keeps the opportunities for our youth (in the sense of Seven Generations) from being compromised by our inaction.

Two-Eyed Seeing asks us to see our strengths, the best in our ways of knowing, while also asking us to respect and celebrate our differences. Two-Eyed Seeing acknowledges the necessity of formal structure yet stresses that it must be preamble to and receptive of new understandings and opportunities. For example, understandings associated with "Spirit of the East" which brings the "gift of newness, of transformation."

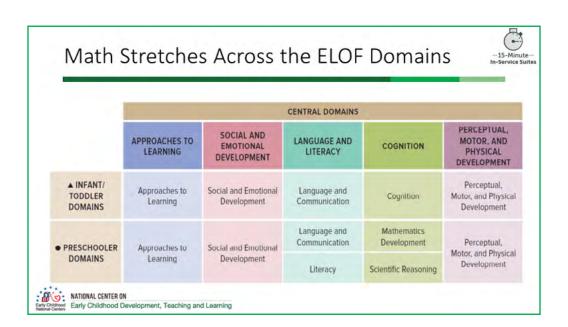
RESOURCE

Integrative Science/Toqwa'tu'kl Kjijitaqnn: The Story of Our Journey in Bringing Together Indigenous and Western Scientific Knowledges:

http://www.integrativescience.ca/uploads/articles/2011-Bartlett-Integrative-Science-Mikmawey-Debert-Chapter-17-proofs.pdf (page 4)

*The term Aboriginal is similar to the word indigenous or native - meaning originating or occurring naturally in a particular place.





SLIDE 4:

The ELOF provides language to help teachers, family child care providers, and home visitors understand child development from a western perspective and support children's development of skills that they will need to succeed in school. Home visitors support families' understanding of child development and promote engaging interactions between parents and their children. It provides a guide for the implementation of effective program and teaching practices that promote strong outcomes for all children, including children with disabilities or suspected delays, children who are dual language learners, and those speaking or learning to speak and write their tribal language.

Head Start Program Performance Standards state that "A program that serves American Indian and Alaska Native children may integrate efforts to preserve, revitalize, or restore, or maintain the tribal language for these children into program services." Our challenge as a program is incorporating cultural traditions, values, and lifeways into our day-to-day work. We need to help children develop the skills identified by the ELOF while supporting a child's cultural identity.

When children learn number recognition and subitizing, they are developing skills in multiple ELOF domains, including approaches to learning and cognition. Children are born primed to explore number ideas as they learn about the world. Number recognition and subitizing skills include counting and cardinality, cognitive self-regulation, and communicating and speaking.

Math Stretches Across the ELOF Domains





- Cognition (Infant/Toddler)
 - · Reasoning and Problem-Solving
 - · Emergent Mathematical Thinking
- Cognition/Mathematics Development (Preschooler)
 - · Counting and Cardinality
- Approaches to Learning
 - · Cognitive Self-Regulation
- · Language and Communication
 - · Communicating and Speaking



SLIDE 5:

For Infants/Toddlers

 Early math skills and concepts that we know are attainable for young children are primarily found in the Cognition domain under the subdomains Reasoning and Problem-Solving and Emergent Mathematical Thinking.

For Preschoolers

• The central domain Cognition is comprised of two more specific domains—Scientific Reasoning and Mathematics Development. Mathematics Development includes the subdomain Counting and Cardinality, which this suite focuses on.

Other areas of development connected to math learning include the Central Domains Approaches to Learning and Language and Literacy.

- For example, the Approaches to Learning domain is about *how* children learn, rather than what they learn. It includes executive function skills that are crucial to math exploration like cognitive self-regulation. Many AIAN cultures engage children in hands-on learning experiences through family and community cultural activities. Elders are an important part of these learning activities.
- When children use language and nonverbal communication, such as eye gaze, raised eyebrows, and gestures, to express interest and talk about what they observe, they use skills found in the Language and Communication domain.

Teaching children in their home language is an important part of connecting math skills to their families, cultures, and developmental goals. You further support children who are learning a tribal language when you describe what the child observes and provide key terms in both their tribal languages and in English.

MATERIALS NEEDED:

- Handout: Head Start Program Performance Standards Support Math, Culture, and Language
- Handout: Number Recognition and Subitizing Stretches Across ELOF Domains

OPTIONAL FOR HS/EHS:

■ Insert Optional Slides 2-5

ADAPTATION FOR HOME VISITORS:

■ Insert HV Optional Slides 1-6



Making It Work is a Resource

We were always trying to integrate culture into the curriculum—the science domain, the math domain, etc. Then we realized that culture should be at the base, serving as the foundation, the building block for curriculum development. It was an amazing paradigm shift! We are now making huge gains in integrating language and culture into our early childhood programming.

—The Red Cliff Early Childhood Center





Early Childhood Development, Teaching and Learning

SLIDE 6:

Read the quote. Briefly introduce the Making It Work (MIW) cycle.

Making It Work is a resource you can use to connect the dots.

Tribes and native communities have always known what research now tells us—when programs integrate culture and language into the classroom's daily activities of and engage families in those activities, children are supported in

- developing a positive sense of self;
- achieving successful child outcomes;
- learning two languages; and
- knowing their community, family, and cultural values, which promotes their resilience and well-being.

RESOURCE

Making It Work: https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/no-search/making-it-work-2017.pdf

MATERIALS NEEDED:

Handout: Making It Work Cycle



SLIDE 7:

Put the slide up and let participants read it. Point out how important language and culture are in everyday activities in the classroom. Keep this in mind as we talk about math learning.

RESOURCE

See page 15 of the Steps and Introduction to Making It Work to learn more about Red Cliff Early Childhood Center's experience integrating language and culture:

https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/no-search/making-it-work-2017.pdf



SLIDE 8:

The Framework for Effective Teaching Practices is known as the House. The House represents five integral components of quality teaching and learning:

- The foundation: Nurturing, responsive, and effective interactions and engaging environments
- The first pillar: Implementing research-based curricula and teaching practices
- The second pillar: Screening and ongoing child assessment
- The roof: Highly individualized teaching and learning
- The center: Parent and family engagement

When connected with one another, they form a single structure—the House Framework—that surrounds the family in the center. As we implement each component of the house—in partnership with parents and families—we foster children's learning and development.



Parent, Family, and Community Engagement

Culturally and linguistically responsive environments can only be created by engaging and partnering with families, Elders, and the community. Establishing a partnership with families and the community is crucial for children's learning and later success in life.





SLIDE 9:

Since the beginning of time, parents, families, and communities have taught their children everything they need to know to become successful members of society.

Children are active learners and take in this kind of information, including cultural knowledge and language, through their physical and social experiences. Programs must develop teaching methods and create learning environments that reflect the cultures and languages of the children they serve.

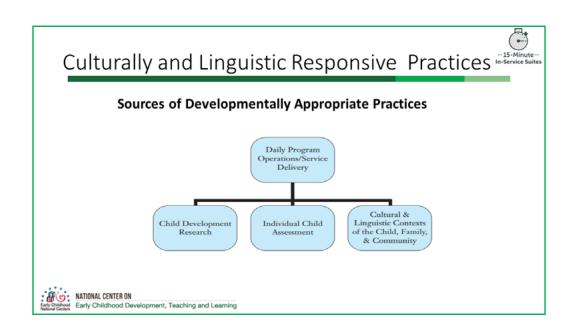
Create culturally and linguistically responsive environments by engaging and partnering with families, Elders, and the community. Establishing a partnership with families and the community is crucial for children's learning and later success in life.

Making It Work suggests ways early childhood staff can intentionally create cultural and language activities for children, families, elders, and community members. It also helps staff align these efforts with their program's Parent, Family, and Community Engagement (PFCE) Framework. The PFCE Framework identifies seven family outcomes, all of which are listed on page 20 in Steps and Introduction to Making It Work: https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/no-search/making-it-work-2017.pdf



SLIDE 10:

- Number recognition and subitizing is an essential part of implementing research-based curricula and teaching practices.
- Children naturally engage with numbers by counting objects in their environment and identifying numbers during circle time. To gain skills in number recognition and subitizing, teachers and home visitors support children and parents by teaching these skills using research-based curriculum and effective practices. Children can learn these skills through engaging environments and interactions during cultural activities at home and in the community.



SLIDE 11:

In AIAN programs, the foundation for learning is children's culture, language, and community involvement. Effective teaching practices incorporate culture and language into everyday activities and the classroom environment.

RESOURCE:

Multicultural Principles for Early Childhood Leaders, Principle 4: Addressing cultural relevance in making curriculum choices and adaptations is a necessary, developmentally appropriate practice. https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/principle-04-english.pdf





Session Objectives



- Explain number recognition and subitizing for young children (the goal)
- Identify the developmental progression for number recognition and subitizing
- List ways to incorporate subitizing into in everyday educational activities, routines, and instruction
- Use the Steps and Introduction to Making It Work Guide to integrate culture and language in lesson plans for math skills



SLIDE 12:

[The screen should be blank] Before you click, pose the following question to your audience:

"How many children did you see in the first screen?" [Take answers]

"Let's look again."

[Click]

"What do you think now? Did you count? If not, how did you know how many? That is number recognition—done quickly, it is called 'subitizing." It is important to note that number recognition (the ability to recognize how many objects are in a group) is NOT numeral recognition (the ability to read numerals). However, we will talk about this more later in the presentation.

- Review the objectives with attendees.
- Ask if anyone has further for suggestions for what they will like to be able to do by the end of this session.

ADAPTATION FOR HOME VISITORS:

■ Insert HV Optional Slides 6-9 to provide a foundation on home visiting practices



SLIDE 13:

READ THE SLIDE

Subitizing is a fundamental skill in the development of children's understanding of number.

REFERENCE:

- Baroody, A. J. 1987. Children's Mathematical Thinking. New York, NY: Teachers College.
- Clements, D. H., J. Sarama, & B. L. MacDonald. 2017. "Subitizing: The Neglected Quantifier." In N. Anderson & M. W. Alibali (Eds.), Constructing Number: Merging Perspectives from Psychology and Mathematics Education. Gateway East, Singapore: Springer.
- Funkhouser, C. 1995. "Developing Number Sense and Basic Computational Skills in Students with Special Needs." *School Science and Mathematics* 95 (5): 236-239.

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Some Examples of Subitizing in the Classroom



- Simple but continuous teaching strategy
 - Use small numbers in everyday talk and cultural storytelling
- You can make a huge difference
 - planned curricular experiences
 - spontaneous experiences





SLIDE 14:

- A simple but continuous teaching strategy makes a huge difference—use small numbers in everyday talk.
- Adults can make huge difference in the spontaneous, informal experiences and interactions that take place between adults and children and between children as they play.
- Everyday talk example:
 - As you incorporate traditional foods into your everyday routines (e.g., snack) and activities (e.g., making blue corn tortillas), use number words and have children tell you how many tortillas are on their plate by subitizing.
 - Extend the examples to number recognition and subitizing from the Pilot Study Program Examples for Making It Work; Blue Corn Tortillas.
 https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/making-it-work-section-03-pilot-program-examples.pdf.
- Storytelling example:
 - Support children's number recognition and subitizing skills, even when verbally sharing cultural stories! For example, if you're talking about two stars, hold up two fingers as you narrate that part of the story.
 - Extend the examples to number recognition and subitizing from the Pilot Study Program Examples for Making It Work; Cultural Storytelling.
 <u>https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/making-it-work-section-03-pilot-program-examples.pdf.</u>



Learning Trajectory



Three Parts:

- 1. Goal
- 2. Developmental Progression
- 3. Educational Activities





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Early Childhood Development, Teaching and Learning

SLIDE 15:

Now that we have reviewed the definition of "subitizing" and had an example, let's learn more about the three parts of learning trajectories.

Learning trajectories have three interrelated parts. First, the goal, which is that children accomplish a mathematical ability in a given domain. Next, children move through levels of predictable *developmental progressions* using *educational activities* intended to encourage a more complex understanding of a concept. We're going to use a learning trajectory to talk about how children learn to recognize numbers and how adults can support them in that development.

REFERENCES:

Clements, D. H., & J. Sarama. 2014. Learning and Teaching Early Math: The Learning Trajectories Approach (2nd ed.). New York, NY: Routledge.

Sarama, J., & D. H. Clements. 2009. Early Childhood Mathematics Education Research: Learning Trajectories for Young Children. New York, NY: Routledge.



SLIDE 16:

Let's examine the first part of the learning trajectory—the goal!



LT Goal for Number Recognition / Subitizing



Children recognize and then subitize (recognize quickly) the number in a group without counting.

"Look! I have three blocks!"





Early Childhood Development, Teaching and Learning

SLIDE 17:

- The goal is for children to recognize the number of objects when they see a group and know how many, without counting!
- Now, in the next few slides, we'll look at what number recognition means!

MATERIALS NEEDED:

Handout: Tips for Families: Number Recognition and Subitizing



SLIDE 18:

[Briefly pass through this slide after simply saying "not this—it's not reading numerals" and then moving on to the next slide to explain what it is].

READ:

NUMERAL recognition is the ability to read numerals, like the 5 you see above. The topic for today is NUMBER recognition—which is the ability to recognize how many objects are in a group. These two concepts are often confused but let's talk more about number recognition so we have a better understanding.



Number Recognition



- Early number recognition is not (yet!) subitizing.
- Subitizing is the *rapid* recognition of numbers without needing to count.
- Children can first recognize the number in small groups, then they get better and *faster* and can subitize.



SLIDE 19:

The ELOF identifies subitizing as a goal for preschoolers, however it is good if children can subitize as early as they can. Before we discuss subitizing in more detail it is important to describe abilities that develop before subitizing—especially for infants and toddlers.

"Subitizing" means "suddenly" or "quickly." When you subitize, you quickly recognize the number in a group. But, in the earliest years, naming numbers is the main goal—even if it takes more than a second or two. Simply naming the numbers is a good start. Therefore, for the youngest children, we say the ability to name small sets is simply "number recognition." Toddlers of course can do this. Once children become increasingly competent at naming numbers easily and effortlessly, we move to subitizing or knowing the number in small groups without counting after only seeing the objects for a very brief time (i.e., seconds).

We will describe higher levels of subitizing as we discuss the developmental progression.

In the next few slides, we will discuss why number recognition is important and highlight research with infants to look at the *earliest* abilities that are the foundation for number recognition and subitizing.



Why Is this Goal Important?



- Number recognition builds upon the earliest developing number sense
 - Infants begin to notice the number of objects in a small group
- Supports learning how to count
 - Cardinality (knowing how many you counted)
- Supports learning arithmetic
 - · Even after accounting for IQ and language



SLIDE 20:

Number sense is children's ability to understand what numbers mean. It is an important early learning outcome in state standards as well as the Common Core and the ELOF.

Number recognition is the earliest developing dimension of number sense. As early as infancy, children develop number sense and start to learn "how many" objects there are, as they become familiar with numbers. Young children will even begin to compare groups of numbers as they develop—getting them ready for subitizing. Children recognize and subitize small numbers to *solve problems*. For example, during play or small group activities, children demonstrate subitizing when they tell their classmates how many objects are in a set, compare numbers, and, later, add or subtract numbers.

Research describes how subitizing helps children learn to count and is particularly important as they begin to understand cardinality (knowing how many you counted). Additionally, subitizing contributes to children's performance on assessments of arithmetic skills above and beyond their general IQ and language skills.

REFERENCES

Sarama, J., & D. H. Clements. 2009. Early Childhood Mathematics Education Research: Learning Trajectories for Young Children. New York, NY: Routledge.

Hannula, M. M. 2005. Spontaneous Focusing on Numerosity in the Development of Early Mathematical Skills. Turku, Finland: University of Turku.



Part 2 of the Learning Trajectory



- 1. Goal
- 2. Developmental Progression
- 3. Educational Activities





SLIDE 21:

Let's study the fine-grained levels of the *developmental progression*, from infants to five years old. Infants? Really? Yes, take a look.



Young Children and Number



- · Infant competencies
- · "Habituation" research





SLIDE 22:

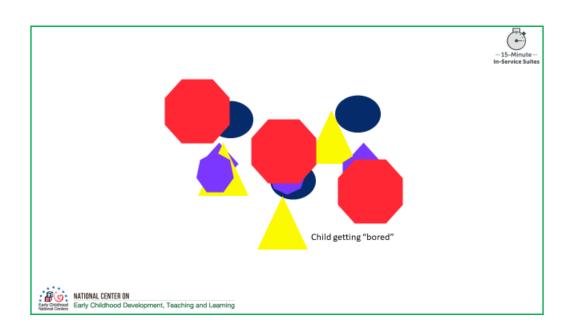
- Even very young children are sensitive to number.
- We know this because of research using what is called the habituation paradigm," where infants lose interest in a series of pictures that differ in some ways, but not others. When an infant has habituated to the pictures, they begin to look at the screen less, and their eyes wander and their breathing becomes more relaxed.

[Click]

- Here is the six-month-old child researchers are studying.
- Instruments, such as those seen here, track electrical signals in the infant's brain (completely painlessly, of course). Scientists have observed that the infant's brain waves move towards sleep as the child habituates.
- Once this child habituates, the researchers change a feature of the display, drawing back the attention of the infant. How does this work?
- Watch what she sees.

NOTES FOR PRESENTER:

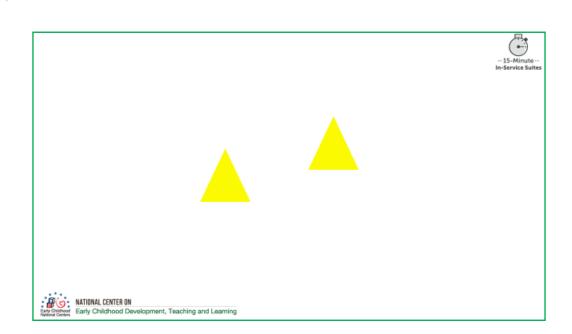
The next few animations show—but far more quickly and fewer in number—the type of display a child might see. You can comment as they pass, that the color, shape, and arrangements all change.



SLIDE 23:

You should see:

- 1. [Click] Three blue circles
- 2. [Click] Three red octagons (Child getting "bored")
- 3. [Click] Two yellow triangles



SLIDE 24:

As the child focuses more on the display, her brain is activated and shows higher levels of activity, and her breathing becomes more rapid. All this suggests to us that she is sensitive to the difference between groups of three and groups of two.

However, this hard-wired ability is not linked to language or concepts (for example, infants do not know 2 is *fewer than* 3) until adults provide experiences that *build* on this natural ability.



Perceptual Subitizing



- · What is it?
 - The ability to "just see" how many objects in a small collection.
- · Let's actually do some perceptual subitizing.
 - · Ready?



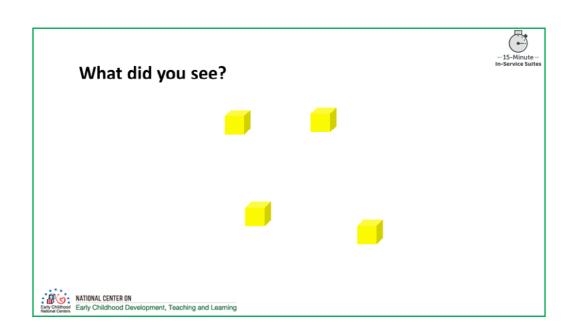
SLIDE 25:

Read "What is it?" text on the slide, then ask:

• What are the types of images we just viewed? (answer: small sets)

Let's do some actual perceptual subitizing.

• Ready? Watch the screen! I will show some blocks for just 2 seconds.



SLIDE 26:

[Let the blocks show and disappear]

[Click]

"What did you see?"

[Click and the blocks reappear] "How did you know?" Discuss.

DISCUSSION POINTS:

• Say that early perceptual subitizing builds on infant sensitivity, *but* relies on experiences, especially naming small numbers!



Conceptual Subitizing

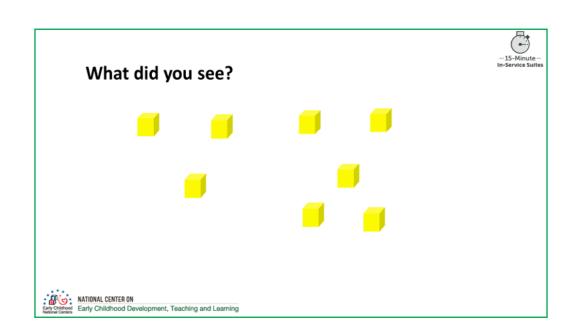


- · What is it?
 - The ability to see the parts of multiple small sets and putting together the whole.
- Now, let's try some conceptual subitizing.
 - · Ready?



SLIDE 27:

- Conceptual subitizing is the ability to see parts of multiple small sets and put them together.
- What are the types of images we just viewed? (answer: small sets)
- Now, let's try some conceptual subitizing.
- Ready? Watch the screen! I will show some blocks for just 2 seconds.



SLIDE 28:

[Let the blocks show and disappear, then advance to the next slide]



What Did You See?



- · How did you know?
- · Think-pair-share!



SLIDE 29:

DISCUSSION: THINK-PAIR-SHARE TO ANSWER THESE QUESTIONS.

What did you see?

How did you know?

Conceptual subitizing builds on perceptual subitizing.

The ability to "see" 8 is conceptual subitizing. Adults "feel" like they see 8, but research shows we really see 3 and 5 and then mentally combine them. That is conceptual subitizing.

Although it is good that so many people have learned from the work of Sarama, Clements, Baroody, and Wheatley about subitizing, many do not know about conceptual subitizing and its importance. Many children can work on conceptual subitizing from 4 years on, and it is important for their mathematical development.

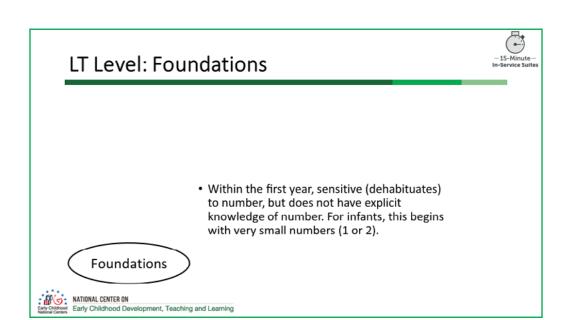
REFERENCES

Baroody, A. J. 1987. Children's Mathematical Thinking. New York, NY: Teachers College.

Clements, D. H. 1999. "Subitizing: What is it? Why teach it?" Teaching Children Mathematics 5 (7): 400-405.

Clements, D. H., J. Sarama, & B. L. MacDonald. 2017. "Subitizing: The neglected quantifier." In N. Anderson & M. W. Alibali (Eds.), Constructing Number: Merging Perspectives from Psychology and Mathematics Education. Springer.

Wheatley, G. H. 1996. *Quick Draw: Developing Spatial Sense in Mathematics*. Tallahassee, FL: Mathematics Learning.



SLIDE 30:

[Go through these next 7 slides very quickly. They simply illustrate levels in the learning trajectory.]

• The first level in the number recognition and subitizing learning trajectory is "Foundations."

[Click]

• Within the first year of life, infants are sensitive to numbers and will dishabituate (get bored and stop looking) when seeing the same repeated numbers. However, they do not know numbers and are not able to correctly identify or recognize them.

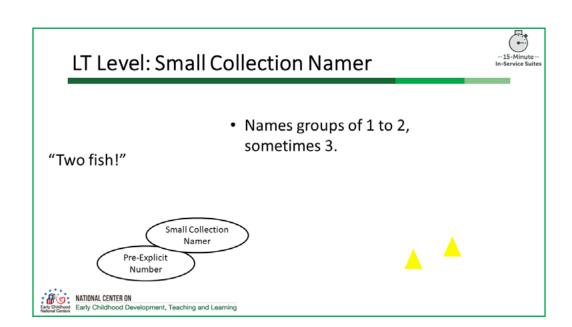
Note: Check with your program's ongoing assessment tool to see how it aligns with the Learning Trajectory levels we will discuss over the next few slides.

MATERIALS NEEDED:

- Handout: Tips for Education Staff: Learning Trajectories of Number Recognition and Subitizing
- Insert Optional Slide 7

ADAPTATION FOR HOME VISITORS:

Handout: Tips for Home Visitors: Learning Trajectories of Number Recognition and Subitizing



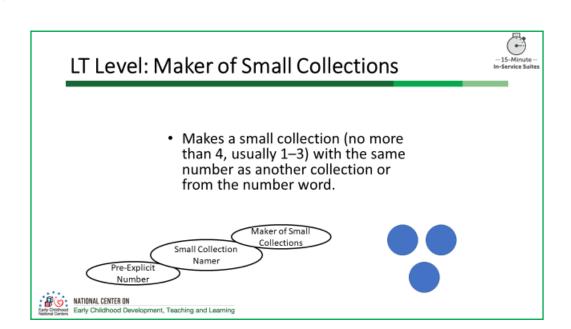
SLIDE 31:

The second level of the learning trajectory is the small collections namer.

[Click]

When shown a group of objects, children at the small collections namer level can *name* groups of 1 to 2, sometimes 3 objects. For example, a toddler may say "2 fish!"

MATERIALS NEEDED:



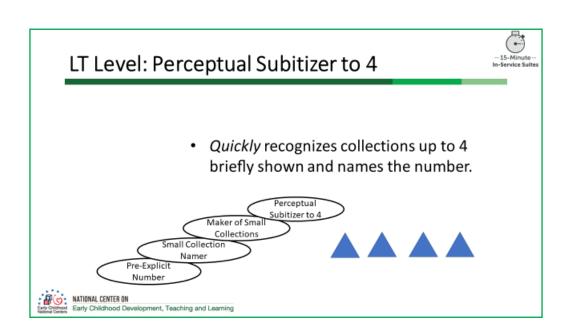
SLIDE 32:

The third level is maker of small collections.

[Click]

Children at the maker of small collections level can *make*, *or produce*, a small collection of objects that have the same number as another collection of objects or by hearing the name of the number. Usually, they can do this with 1-3, and no more than 4, objects.

MATERIALS NEEDED:



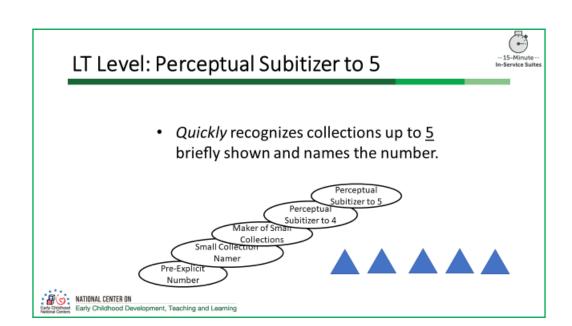
SLIDE 33:

The fourth level is perceptual subitizer to 4.

[Click]

Children at the perceptual subitizer to 4 level can *quickly* recognize small collections of up to 4 objects when briefly shown. They can also name the number without counting objects.

MATERIALS NEEDED:



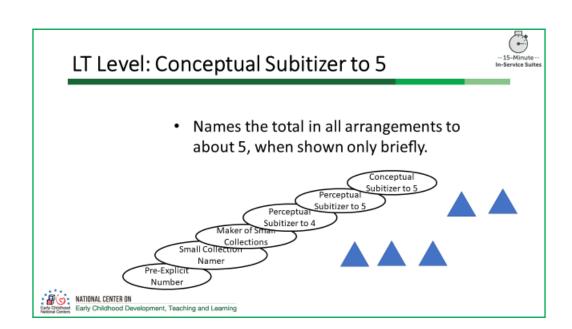
SLIDE 34:

The fifth level is perceptual subitizer to 5.

[Click]

Children at the perceptual subitizer to 5 level can *quickly* recognize small collections of up to 5 objects. when briefly shown; they can also name the number without counting objects.

MATERIALS NEEDED:



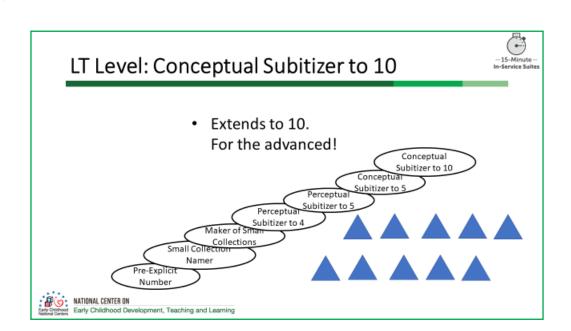
SLIDE 35:

The sixth level is conceptual subitizer to 5.

[Click]

Children at the conceptual subitizer to 5 level can quickly name the total of objects, up to 5, in *all different* arrangements when shown briefly (such as briefly seeing a group of 2 objects and a groups of 3 objects and knowing that there are 5 total objects); They can also name the number without counting objects.

MATERIALS NEEDED:



SLIDE 36:

The seventh level is conceptual subitizer to 10.

[Click]

Children at the conceptual subitizer to 10 level can *quickly* name the total of objects, up to 10, using groups. They can also name the number without counting objects.

MATERIALS NEEDED:



Part 3 of the Learning Trajectory



- 1. Goal
- 2. Developmental Progression
- 3. Educational Activities





SLIDE 37:

How can teachers, family child care providers, home visitors, and parents support children's growth of number recognition and subitizing?

Let's start with this: Do we do a good job now?



Math Language



- When children make a math utterance, teachers:
 - 60% of the time ignore it
 - only 10% of the time respond mathematically

This has important implications for children, particularly children speaking or learning their tribal language.



SLIDE 38:

What does research say about the way early childhood teachers respond to children speaking about math suggests? Ask participants what they think the implications are of this research finding.

REFERENCE

Diaz, R. M. 2008. "The Role of Language in Early Childhood Mathematics: A Parallel Mixed Method Study." FIU Electronic Theses and Dissertations. <a href="https://digitalcommons.fiu.edu/cgi/viewcontent.cgi?referer=https://scholar.google.com/&https://schol

OPTIONAL MATERIALS:

Insert Optional Slide 14

ADAPTATION FOR HOME VISITORS:



Support Subitizing for AIAN Children



- 1. Choose culturally meaningful and familiar materials
- 2. Connect math terms to child's tribal language
 - · Learn if the child can subitize in their tribal language?
 - · Embed language of child in activity
 - · Use one or two words and phrases
- 3. Multiple representations
- Emerging competence versus struggles with expressive vocabulary
- 5. Tiered levels of questions





SLIDE 39:

Children learning a tribal language will benefit from math experiences that represent their language and culture**.

- 1. Choose materials that give children clues about what they are supposed to be learning. If children in your learning environment are famliar with setting the table for snack or sorting the blocks before they go on the shelf, finding ways to incorporate what children already know with something you would like for them to learn is the best way to build connections between everyday routines and math topics—no matter what language children are proficient in. For example, counting out the cups for each table before snack will eventually help children subitize and learn numerical vocabulary. (https://www.naeyc.org/resources/pubs/tyc/oct2017/make-math-meaningful-diverse-learners)
- 2. Connect to children's languages.
 - a. Find out if the child is able to subitize or count in his or her tribal language. Use index cards to write down frequently used words that children may speak in their tribal language.
- 3. Accept multiple representations as responses to tasks—for example, fingers, marks on paper, etc.
- 4. Consider tiered levels of questions in order to assess child understanding (Krashen & Terrell, 1983; Tabors, 2008).
 - a. Non-verbal questions (closed): "Show me...", "Point to..."
 - b. Telegraphic/Formulaic (closed): Yes/No, Either/Or
 - c. Productive (open-ended): "How do you know?", "What is another way to show me X?"

**Make efforts to preserve, revitalize, restore, or maintain children's tribal languages (HSPPS 1302.36). Encourage children to use their tribal languages and provide meaningful math opportunities that represent their cultures.

Discussion: If you or the children in your program speak (or are learning) a tribal language as well as English, do you consider yourself or the children in your program dual language learners? What about non-tribal children in your programs who speak a home language and English? Do you consider them dual language learners?

MATERIALS NEEDED:

- Handout: Activities for Education Staff: Fantastic Five
- Handout: Tips for Education Staff: Supporting Families with Math Learning
- Handout: Activities for Families: Help Your Child Learn

ADAPTATION FOR HOME VISITORS:

- Insert HV Optional Slide 11
- Handout: Tips for Home Visitors: Supporting Families with Math Learning
- Handout: Activities for Families: Help Your Child Learn



SLIDE 40:

Here's a basic activity called "Snapshots." Show a set for 2 seconds or less, and then hide it. Ask children to say how many they saw. Use images that are culturally relevant to children so that they may easily find the words to express what they are seeing. For example, if you use images of words you're practicing in children's tribal language, you can get a "two-for-one" activity—children can practice subitizing *and* hearing tribal names and seeing objects that are culturally relevant to them!

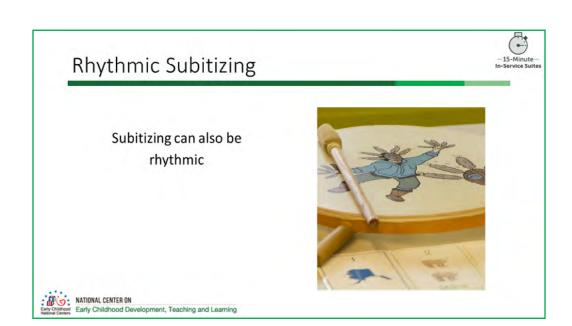
- How would you need to alter these cards to play this game?
 - Remove or hide the numerals (written letters) when you play Snapshots. You can use the cards with both the images and the numerals at another (or later) time.

MATERIALS NEEDED:

- Handout: Learning Activity for Education Staff: Subitizing Throughout the Day
- Handout: Activities for Families: Brainstorming

ADAPTATION FOR HOME VISITORS:

- Handout: Learning Activity for Home Visitors: Brainstorming
- Handout: Activities for Families: Brainstorming



SLIDE 41:

Subitizing can also be rhythmic during singing or music time.

For example, rhythmically beat a drum for a few beats and have children tell or show you on their fingers how many beats they heard.

Extend the example to number recognition and subitizing. You can see an example of this in the Pilot Study Program Examples for Making It Work, Pow Wow: https://eclkc.ohs.acf.hhs.gov/sites/default/files/pdf/making-it-work-section-03-pilot-program-examples.pdf)

Supporting Children with Suspected Delays or Identified Disabilities



- Subitizing is especially important for the mathematical development of children with special needs.
- Follow the learning trajectory with patience!
- Use number names all day, naturally but intentionally.
- Play a lot of dice and domino games.





SLIDE 42:

Given it's foundational and critical role in children's initial number sense development, it is especially important for the mathematical development of children who may have suspected delays or identified disabilities (Baroody, 1986).

Following the learning trajectory (described in the ELOF and here in the PPT) back to the beginning if necessary, with informal and intentional experiences over an *extended time*, is critical. Using small (and slowly increasing) numbers in everyday talk and storytelling is particularly important in informal experiences. Simply saying, "You ate *three* berries!" and so forth, throughout the day, does not take away time from anything else, and is a golden opportunity to help all children develop a sense of numbers. Dice, domino, and board games that use die are fun, simple examples of intentional experiences that research says helps children build number sense in a variety of ways. Play traditional games, such as those found through the International Traditional Games Society: https://www.traditionalnativegames.org/games-flip-book.

REFERENCES:

Clements, D. H., J. Sarama, & B. L. MacDonald. 2017. "Subitizing: The Neglected Quantifier." In N. Anderson & M. W. Alibali (Eds.), Constructing Number: Merging Perspectives from Psychology and Mathematics Education. Gateway East, Singapore: Springer.

Funkhouser, C. 1995. "Developing Number Sense and Basic Computational Skills in Students with Special Needs." *School Science and Mathematics* 95 (5): 236-239.

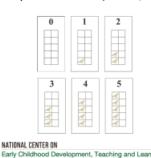
International Traditional Games Society. 2016. Native Games of Montana Tribes. https://www.traditionalnativegames.org/games-flip-book



Children with Disabilities



 Work toward use of fives and tens frames, a powerful representation (Flexer, 1989)



- For those with perceptual issues, such a visual impairment:
 - use manipulatives that can be held, and
 - · include lots of rhythmic subitizing.

SLIDE 43:

Researchers who work with typically and atypically developing children promote the use of the five and ten frame. Organizing the dots on the grids (seen here) provides a simple visual image that children can associate with each number—supporting perceptual subitizing development. These mental images also help children as they learn to partition small numbers into different combinations. For example: 0+5=5, 1+4=5, 2+3=5, and so forth. When they can do this quickly, this is *conceptual* subitizing.

The five and ten frame can and should be made for children with perceptual delays. If children are visually impaired, use manipulatives they can hold—such as beads on a string or sticks—or use rhythmic subitizing.

REFERENCES:

Clements, D. H., & J. Sarama. 2014. Learning and Teaching Early Math: The Learning Trajectories Approach (2nd ed.). New York, NY: Routledge.

Clements, D. H., J. Sarama, & B. L. MacDonald. 2017. "Subitizing: The Neglected Quantifier." In N. Anderson & M. W. Alibali (Eds.), *Constructing Number: Merging Perspectives from Psychology and Mathematics Education*. Gateway East, Singapore: Springer.

Flexer, R. J. 1989. "Conceptualizing Addition." Teaching Exceptional Children 21 (4): 21–25.

Flexer, R.J. 1986. "The Power of Five: The Step Before the Power of Ten." The Arithmetic Teacher 34 (3): 5-9.

Shih, J., W. R. Speer, & B. C. Babbitt. 2011. "Instruction: Yesterday I Learned to Add; Today I Forgot." In F. Fennell (Ed.), *Achieving Fluency: Special Education and Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Van Luit, J. E. H. 1994. "Dealing with Learning Difficulties Concerning Addition and Subtraction: Due to or Despite the Little Person?" In J. E. H. Van Luit (Ed.), *Research on Learning and Instruction of Mathematics in Kindergarten and Primary School.* Doetinchem, Netherlands: Graviant.



SLIDE 44:

- Number recognition is the earliest developing number sense. Support children as they learn how to count and eventually working out math problems.
- Subitizing is the rapid recognition of numbers without needing to count.
- Perceptual subsidization begins in infancy. As children get older, they can conceptually subitize—recognize the total amount of objects in different grouping—without counting!
- When teachers, home visitors, and parents use small numbers in everyday language, this helps support children's number recognition and subitizing.
- Work with families in both center- and home-based settings to encourage math learning at home.
- To support children who are tribal language learners, provide multiple representations of numbers to help build their understanding.
- Embed children's tribal language into math activities to help them understand number recognition and subitizing.
- Math is a part of a child's everyday environment. Allow them to explore and be there to intentionally support their math development
- Making It Work is a resource that helps you learn and teach math skills through the integration of tribal languages and cultures.

MATERIALS NEEDED:

- Helpful Resources
- Tools for Supervisors
- Insert Optional Slide 15